Automatic Characterization of Large Arrays of Superconducting Far-Infrared Detectors for Astronomy

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The observations of the Universe in the Far Infrared (FIR) spectrum are vital to astronomers and require the use of instruments based on large arrays of cryogenic sensors like the Microwave Kinetic Inductance Detector (MKID). A MKID array has more than 1000 detectors displaced in frequency that need to be characterized. The currently implemented solution uses the Second Derivative of the frequency spectrum's amplitude. Observations showed that this method is highly susceptible to high frequency noise, making it too dependent on operators to select a correct threshold. The proposed solution uses the Continuous Wavelet Transform (CWT) to improve the signal to noise ratio of the spectrum, allowing for more stable thresholding to be performed. The algorithms were compared using real spectrum scans and simulated resonances. With an optimal configuration for both algorithms, the CWT detected 4% more resonances in the real data and simulated results prove that it can operate successfully in the presence of close to 40% more high frequency noise than the current solution. The improvements to the resonance characterization result in more efficient use of the MKID arrays under a greater variety of conditions. This allows for better FIR observations, using instruments with higher resolutions.
Declaration

I hereby certify that this report constitutes my own product, that where the language of others is set forth, quotation marks so indicate, and that the appropriate credit is given where I have used the language, ideas, expressions or writings of another.

I declare that the report describes original work that has not previously been presented for the award of any other degree of any institution.

Signed,

_______________________
Valentin Gabarov
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LIST OF TABLES

Table 1 Comparison between MATLAB and Python 2.7 algorithms using the Second Derivative method. Statistics show are not dependent on the circle fitting procedure. ................................................................. 31
Table 2 Statistical comparison between the characterized detects of the Second Derivative and CWT algorithms. .................................................................................................................. 35
Table 3 Original data set algorithm performance ................................................................. 36
Table 4 Load Temperature Variation (LT012) Detection count comparison. .............................. 39
Table 5 Base Temperature Variation (K123) Detection count comparison using calibration for LT028 Al Foil -42dBm. .................................................................................................................. 39
Table 6 Eye detections offset map, with respect to readout power and base temperature. .......... 40
Table 7 Number of detections by the Second Derivative and CWT algorithms, with respect to the base temperature and readout power of array K123. ................................................................. 41
Table 8 Input signal parameters and range sizes used during the Monte Carlo Simulation .............. 43
Table 9 Second Derivative Algorithm calibration variation during the Monte Carlo Simulation .......... 43
Table 10 CWT Algorithm calibration variation during the Monte Carlo Simulation ..................... 43
Table 11 Time comparison between CWT and 2nd Deriv. over the K123 Precision and Recall Tests ........ 45
Table 12 Structure of a MKID readout electronics’ sweep file. The left most column is the LO frequency, which is used to relate the bins of the readout to a real frequency. The red rectangles represent a single point in the frequency spectrum. The first value is the bin number, second is the real $S_{21}$ transmission and the third is the imaginary $S_{21}$ transmission. Using the LO frequency, the bin number, and configuration stored in another file, the spectral information needs to be reconstructed. ................. 65
LIST OF FIGURES

Figure 1 Atmospheric absorption limits earth observation in the infrared spectrum. In order to obtain higher accuracy and wider range of wavelengths a space observatories are required. Source: NASA 2008 Public Domain .................................................................................................................................................. 12

Figure 2 A graphical representation of the emission spectrum from a protoplanetary disk and the telescopes that can be used to probe different regions of the disk. Source: Dullemond & Monnier (2010) .................................................................................................................................................. 13

Figure 3 Centaurs A in visible (left) captured using the Hubble Space Telescope and far-infrared (right) captured using the Herschel Space Observatory’s SPIRE instrument. Source: NASA, ESA & M. Rejkuba (left), ESA, Herschel, PACS, SPIRE, C.D. Wilson MacMaster University Canada ................................................................. 13

Figure 4. Principle of operation of the MKID. At low temperatures and high frequency in a superconductor the property known as superconducting kinetic inductance becomes significant. When a photon hits the detector Cooper pair are broken, which results in a change of inductance. The change in inductance shifts the resonance of the circuit, which is being measured. Source: SRON Jochem Baselmans .................................................................................................................................................. 15

Figure 5. MKID Array frequency division multiplexing. The MKIDs are represented by the combination of variable resistor and inductor in the blue box. Coupled with the capacitors a separate resonant circuit is created for every detector tuned to a different frequency. Up to a thousand detectors can be read out using only the two signal lines represented here above and below the resonant circuits. [12]...................... 15

Figure 6 Readout Electronics for the KID based instrument. In the Digital to Analog Converter waveforms are generated and different frequencies, which are called tones. The tones are upscaled to higher frequencies and pass through the superconductive detectors inside a cryostat. Afterwards the output signal is compared to the input and a transmission spectrum is generated. Source: SRON Jochem Baselmans .................................................................................................................................................. 16

Figure 7. Example of second derivative detection algorithm. The blue line represents the original frequency spectrum of the array, the green line is the second derivative of that spectrum. The red line is the set threshold and the red points are the detected resonances. ................................................................. 17

Figure 8 Example of a circle fitted and centered resonance in LT028 Al Foil -42dBm. The blue dots represent the resonance, with the red being the resonant frequency. The green circle is the calculated fit. .................................................................................................................................................. 20

Figure 9 Example of a double resonance fitted incorrectly as one, in the LT028 Al Foil -42dBm. The blue dots represent the resonance, with the red being the resonant frequency. The green circle is the calculated fit. .................................................................................................................................................. 20

Figure 10 Flow charts describing the steps used for the characterization of a MKID array starting from the raw spectral data and ending with individual resonance parameters. The algorithm was obtained from Second Derivative method, by dividing it into functional blocks................................................................. 21

Figure 11 Deconstruction of “LT028 Al Foil” amplitude using the Discrete Wavelet Transform. The original signal is shown on the top. Left hand side represents the result of the low-pass filtering and the right hand side of the high pass filtering. The DWT was performed using a Haar wavelet. ................................................................. 28

Figure 12 Graph showing a Mexican Hat Wavelet at different scales. [26] ................................................................. 29
Figure 13 Top: Results of a CWT Bottom: the amplitude of 7 resonances. The data set is LT028 Al Foil - 42dBm readout power resulting in low noise and well defined resonances. ................................................................. 30
Figure 14 Top: Results of a CWT Bottom: the amplitude of 7 resonances. The data set is LT028 Al Foil - 50dBm readout power resulting in higher noise and shallower resonances. ................................................................. 30
Figure 15 Top: Results of a CWT Bottom: the amplitude of 7 resonances. The data set is LT028 Al Foil - 30dBm readout power resulting in high frequency artifacts and highly asymmetrical resonances. These MKIDs are being overdriven. ........................................................................................................... 30
Figure 16 Q factors obtained using MATLAB. The readout power stated and difference is because of cryostat calibration is used. ........................................................................................................................................ 31
Figure 17 Q factors obtained using Python 2.7 Second Derivative algorithm. ................................................................................................................................. 31
Figure 18 Example operation of the CWT algorithm, between 7.656 GHz and 7.661 GHz of MKID array LT028 Al Foil at -42dBm readout power. Top: Amplitude of input frequency spectrum Middle: Extracted CWT areas defining the resonances. Bottom: Outline of the resonances in the CWT above the 5 times standard deviation threshold (blue lines) and the minima (red dots)......................................................................................... 33
Figure 19 Full characterization of the LT028 Al Foil -42dBm MKID array, using the CWT algorithm. ........ 35
Figure 20 Amplitude overview scan of LT028 Al Foil at -50dBm power level, interpolated using CWT..... 36
Figure 21 Comparison of KID finding algorithms on the amplitude scan of LT028 Al Foil data set at high frequencies, interpolated using CWT. ......................................................................................................................... 37
Figure 22 Comparison of KID finding algorithms on the amplitude scan of LT028 Al Foil data set comparing the two algorithms with the eye detection. Interpolation was done using CWT. ................. 37
Figure 23 Precision and Recall results of the LT028 Al Foil VNA power sweep............................................... 38
Figure 24 Example of shift between different temperature and readout power in the interpolated amplitude of K123................................................................................................................................. 40
Figure 25 Precision and Recall comparison between the Continuous Wavelet Transform and Second Derivative detection algorithms relative to the readout power and base temperature of a Microwave Kinetic Inductance Array K123. ......................................................................................................................... 42
Figure 26 Simulated signal consisting of two resonances, used as input for the Monte Carlo simulation. 43
Figure 27 Number of successful detections of the Second Derivative algorithm Monte Carlo simulation, with respect to the level of high frequency noise in the input.............................................................. 44
Figure 28 Number of successful detections of the CWT algorithm Monte Carlo simulation, with respect to the level of high frequency noise in the input........................................................................ 44
Figure 29 Number of successful detections of the Second Derivative algorithm Monte Carlo simulation, with respect to the distance between the two resonances in the input.............................................. 44
Figure 30 Number of successful detections of the CWT algorithm Monte Carlo simulation, with respect to the distance between the two resonances in the input.................................................. 44
Figure 31 Flow chart of the pre-processing performed in the Second Derivative algorithm. The input complex information is scaled with respect to its maximum value as well as the delay in the phase is removed. ......................................................................................................................... 53
Figure 32 Flow chart of the pre-processing performed in the Continuous Wavelet Transform algorithm. The input complex information is scaled with respect to its maximum value.................................................. 53
Figure 33 Flow chart of the signal to noise ratio improvement performed in the Second Derivative algorithm. This is the defining step of the algorithm, responsible for differentiating the signal from the noise......................................................................................................................... 53
Figure 34 Flow chart of the signal to noise ratio improvement performed in the Continuous Wavelet Transform algorithm. Separating the signal and noise is performed here for CWT. ................................. 53
Figure 35 Flow chart of the resonance detection performed in the Second Derivative algorithm. In this step a threshold is applied to the second derivative. All sequential areas above a minimum size are considered areas of interest. ................................................................................................................................. 53
Figure 36 Flow chart of the resonance detection performed in the Continuous Wavelet Transform algorithm. In this step sequential areas above a minimum x and y size are considered as areas of interest & paired with previous data. ................................................................................................................................. 53
Figure 37 Flow chart of the system ripple removal performed in the Second Derivative algorithm. The areas below the second derivative are reduced on their edge by a set value. The resulting signal is smoothed and interpolated at the gaps, where the resonances were. Dividing by the original data results in a signal clean of system ripple. ................................................................................................................................. 54
Figure 38 Flow chart of the system ripple removal performed in the Continuous Wavelet Transform algorithm. The average size of the areas of interest found is added to each side of the areas. The interpolation is performed in the same way as the Second Derivative method, with the signal outside the areas.................................................................................................................................................. 54
Figure 39 Flow chart of the circle fitting performed in the Second Derivative algorithm. In this step a least square circle fit is performed in the complex plane, using a preset number of points around every resonant frequency found. From the result a calibration parameter is generated. .................................................. 54
Figure 40 Flow chart of the circle fitting performed in the Continuous Wavelet Transform algorithm. Using the local minimums in the CWT outline, the algorithm looks for resonances. If more than one are found, the area of interest is split at the local maximum of the outline between them. Circle fitting is performed on a portion of every area of interest. The complex data is corrected on the spot. ........... 54
Figure 41 Flow chart of the post processing performed in the Second Derivative algorithm. Using the corrected phase each MKID is Characterized producing the end results in green................................. 54
Figure 42 Flow chart of the post processing performed in the Continuous Wavelet Transform algorithm. Using the new phase each MKID is Characterized producing the end results in green................................. 54
Figure 43 Interpolation example of CWT on the original data. ................................................................. 56
Figure 44 Interpolation example of CWT on the Space KIDs. ................................................................. 56
Figure 45 High frequency noise in L20 amplitude. .................................................................................... 56
Figure 46 Asymmetrical resonances in L20 amplitude. ............................................................................ 56
Figure 47 Interpolated amplitude of LT012 with a 77 K load. ................................................................. 58
Figure 48 Interpolated amplitude of LT012 with a 300 K load. ............................................................. 58
Figure 49 Example of false positives in the CWT and false negatives in the Second Derivative. Array
LT012 at -42dBm and load temperature of 300K. ......................................................................................... 58
Figure 50 CWT false positives example in interpolated amplitude of K123 220mK -42dBm. ...................... 59
Figure 51 Example of many false positives in the K123 340mK -42dBm data set. ..................................... 60
Figure 52 Amplitude overview of K123 at 340mK and -42dBm ............................................................... 60
Figure 53 Amplitude overview of K123 at 400mK and -42dBm ............................................................... 61
Figure 54 Example of noise in interpolated amplitude of K123 at 400mK and -42dBm. ............................ 61
Figure 55 Amplitude overview of K126 interpolated using CWT ............................................................ 62
Figure 56 Example of low Q factor resonance in the interpolated amplitude of K126 ......................... 63
Figure 57 Example of a stitching error in the interpolated amplitude of K126 ......................................... 63
Figure 58 Examples of false positive CWT detections in the K126 interpolated amplitude ..................... 63
Figure 59 Example of overlapping points between individual bins and the LO frequency groups. In the amplitude overlapping the divergences between points of neighboring bins is small, however the gap between frequency groups is significant. In the phase both types of overlaps have a considerable difference between them as well as discontinuities occur during phase wrapping when it passes $\pi$.

Figure 60 Pure Amplitude and Phase plot overview of the frequency spectrum of array K125 obtained using the new readout electronics. The 5 obvious colored arrays are different LO frequency zones in the readout. The first starts at 3950.01 MHz and the second at 4773.99 MHz. The space in between is covered by a 103 LO frequencies with a step of 10 kHz. The process is repeated until the full spectrum is covered.

Figure 61 Final product of the stitching pre-processing. The smallest difference between the points of two overlapping bins is taken. The amplitude of the second bin is multiplied by the difference, and in phase the offset is added. The resulting signal is non-uniform with steps no bigger than the LO frequency step at the stitching points.
1 **RATIONALE**

This chapter aims to provide background information, related to the research. It starts with astronomy’s need for far-infrared detectors and reaches the currently faced technological challenges that result in this project. Finally the main research question and clarifying sub-questions are defined.

1.1 **Scientific Goal**

The Netherlands Institute for Space Research Groningen (SRON) has two locations: Utrecht and Groningen. At the Groningen location the main focus is on the development of instruments for observing the cold Universe. The Cold Universe is defined in this case as objects in the temperature range between 10 and 2000 degrees Kelvin, the source of infrared light and submillimeter radiation. The far infrared (FIR) detectors, which are the focus of this research project detect wavelengths between 100 and 1000 microns [1]. Information contained in these wavelengths is required by astronomers in order for them to obtain more information on the evolution of galaxies and celestial bodies.

The formation and structure of galaxies are studied from the earliest epochs, when the first stars were formed in the infant Universe, to the present day, where the fossils from those epochs can be found. The expansion of the Universe has caused those earliest galaxies to be redshifted into the FIR spectrum. They physics of galaxy evolution can be theorized by performing large scale surveys and gathering statistical information on the redshifted galaxies. The Atacama Pathfinder Experiment (APEX) is a telescope capable of performing such surveys from Earth [2]. The APEX telescope provides the optics needed, but the sub-millimeter instrument required for FIR studies is called A-MKID.

A-MKID is a camera based on the latest available technologies. It operates in two frequency bands simultaneously, in the atmospheric windows around 850µm and 350µm, and offers in total about 25000 pixels to the user. The camera uses Microwave Kinetic Inductance Detector (MKID) arrays, since they show highest sensitivity and their production on large wavers with high yield has been demonstrated [3]. Figure 1 shows that the ground based uses of FIR wavelengths is limited to small dynamic windows, available only at high altitudes like the APEX site.
FIR instruments like SAFARI used on space observatories such as the proposed SPICA can provide insight into the formation of new stars and formation of exoplanets, through their Spectral Energy Distribution (SED) [4]. Different regions of a Young Stellar Object (YSO) emit at different wavelengths, shown in Figure 2. Proto-stars are heavily embedded in dust and gas and are thus invisible at optical wavelengths. Studying the protoplanetary disks in multiple wavelengths using different instruments allows astronomers to validate their theories on planet formation. The FIR detectors are required in order to probe the outer, cooler regions of the disk [5] as well as galaxy dust clouds seen in Figure 3.

![Figure 2](image_url)  
*Figure 2 A graphical representation of the emission spectrum from a protoplanetary disk and the telescopes that can be used to probe different regions of the disk. Source: Dullemond & Monnier (2010)*

![Figure 3](image_url)  
*Figure 3 Centaurs A in visible (left) captured using the Hubble Space Telescope and far-infrared (right) captured using the Herschel Space Observatory’s SPIRE instrument. Source: NASA, ESA & M. Rejkuba (left), ESA, Herschel, PACS, SPIRE, C.D. Wilson MacMaster University Canada (right)*
1.2 Far Infrared Detectors

Every astronomy observatory, on the ground or on a satellite, has a number of instruments that are designed for different frequencies, resolutions, sensitivities etc. The development of the instrument includes the design, building and testing of all components required for the instrument to operate as a black box. These components can be calibration equipment, cooling liquid pumps, filters, lenses, readout electronics and so forth. These components are created by a great variety of experts ranging from mechanical engineers to applied physicists [6] [7].

The instruments developed at the Groningen location of SRON use detectors, which can operate on a variety of different physics principles, in large arrays up to several thousand pixels. Although they differ in their operation they all require cryogenic temperatures, too low for Charged-Coupled Devices. The temperatures depend on the specific type of detector and in some cases can go as low as 25mK [1].

The cryogenic temperatures present design restrictions concerning the amount of wiring that can be used as signal lines. In order to maximize the number of detectors that can be read, with a small amount of wiring, frequency division multiplexing (FDM) is used [4]. FDM is the root cause of the problem that this project aims to solve. FDM has been used in many fields for a long period of time [8], one of the first being the radio receiver. The user tunes in to a specific frequency and listens to only one channel, and can freely change channels. The main reason why FDM is simple to implement in the case of the radio is the fact that every channel has a predetermined position, with a dedicated empty frequency space around it [9].

When working with superconducting detector arrays, of a thousand pixels and more, the FDM situation becomes more complex. This is due to the combination of high frequencies, higher level of integration and limits of lithographic precision [10].
The project focuses on the Microwave Kinetic Inductance Detector (MKID) [10], however its results can benefit other types of detector arrays, like the ones using Transition Edge Sensors (TES) [4]. The TES is a thin composite layer of Titanium and Gold operated at the superconducting to normal transition temperature. This allows it to convert small changes in temperature into large changes of electrical resistance. In the MKID on the other hand, photons break up Cooper pairs into quasiparticles as shown on the left side of Figure 4. This results in changes in inductance that are measured.

![Figure 4. Principle of operation of the MKID. At low temperatures and high frequency in a superconductor the property known as superconducting kinetic inductance becomes significant. When a photon hits the detector Cooper pair are broken, which results in a change of inductance. The change in inductance shifts the resonance of the circuit, which is being measured. Source: SRON Jochem Baselmans](image)

The MKID instrument needs to implement a small amount of electronics at its coldest stage, which results in a fairly simple resonant circuit. The detector is a $\frac{1}{4}$ wavelength resonator, which can be modelled by a lumped element equivalent circuit as is Figure 5. The circuit of every detector is designed to resonate at a specific frequency equally spaced from other detectors. The detector ship is manufactured on a sapphire wafer and in reality each element on it has a certain fabrication tolerance. In the case of the MKID, the resonator is defined by its length, width and thickness, the most sensitive of which is the thickness measured in fractions of a micron. Variations in these parameters cause the resonant frequency of the detectors to shift from the designed value. Thus, every manufactured MKID (as well as TES LC filters) array must be individually measured [11].

![Figure 5. MKID Array frequency division multiplexing. The MKIDs are represented by the combination of variable resistor and inductor in the blue box. Coupled with the capacitors a separate resonant circuit is created for every detector tuned to a different frequency. Up to a thousand detectors can be read out using only the two signal lines represented here above and below the resonant circuits. [12]](image)
The measuring of a detector array is performed by generating a signal composed of frequency tones. For the first test the tones are at equally spaced frequencies, covering the entire frequency space, which is referred to as a wide sweep. For the wide sweep each frequency tone has the same amplitude, which is called readout power. This signal is passed through the array, each detector in the array will affect a small frequency space and then the exiting signal is measured by a spectral analyzer. The procedure is performed using a setup like the one in Figure 6. The output of the analyzer is the transmission data set used to find the resonances called $S_{21}$ (Transmission from port 1 to port 2). Detectors will be different depending on how close they are to a frequency tone. Once the location of every one is known the tones can be placed exactly on the resonant frequencies of the detectors, resulting in the best possible signal to noise ratio [11].

Although the biggest shift in the resonant frequency of a KID comes from the manufacturing, it is affected by other conditions. Those conditions are read out power, array base temperature, radiation absorption etc. [12]. All this creates the need to characterize the arrays by finding the resonances of all detectors and keeping track of their identity during parameter changes. This process needs to function with the presence of a low frequency ripple caused by the cabling and high frequency noise.
1.3 Current Solution

The current algorithm for detecting the resonances in MKID arrays takes the second derivative of the frequency spectrum of the array as the main method for detecting the resonances. It involves pre filtering techniques performed on the frequency spectrum and post filtering once the detector’s resonant frequencies have been found. The biggest increase in signal to noise ratio is accomplished by the second derivative, so it is considered the algorithm’s core. Then the decision of what is regarded as a signal is made using a user defined threshold. During this process more weight is given to detecting all resonances over the detection of false positives, by selecting a lower threshold as shown in Figure 7. This is done because once the location of the MKID resonances is known, a circle fitting is performed in the complex plane for each MKID. In order to fit each resonance to a circle both amplitude and phase information is used so it becomes easier to distinguish between true detectors and false positives. Once a circle is fitted over an MKID its value is obtained by measuring the angle between the baseline (no absorption) and the current position of the resonance. The changes in phase are greater than the changes in amplitude when an MKID absorbs a photon [12]. The circle fitting can be observed on the right hand side of Figure 4.

![MKID Finding](image)

*Figure 7. Example of second derivative detection algorithm. The blue line represents the original frequency spectrum of the array, the green line is the second derivative of that spectrum. The red line is the set threshold and the red points are the detected resonances.*
1.4 Areas to Improve

The current solution is functional; however, it does have some significant drawbacks. Primarily, the threshold for the resonance detection is selected by a human operator. This means that the decision of what is considered a MKID resonance and what is not is done by the human, while still being prone to errors. Manually going through a data set gives the best possible results, but can take more than 4 hours depending on the number of resonances. A desired improvement would be for this to be performed by the software processing the information using an algorithm. Furthermore, it is possible that using an alternative to the second derivative algorithm could increase the signal to noise ratio (SNR) even further, or improve the resolution in order to differentiate between or identify overlapping resonances. This would make the threshold selection more reliable. So it is desired to explore alternatives to the current solution and compare them to it, in order to improve the MKID finding procedure to a point where it is independent of user input [10].

The focus of this project is on alternatives to the finding algorithm. This includes the steps after the preprocessing of the frequency spectrum until the circle fitting at the post processing stage. The finding algorithm can be divided into two subsections, improving the signal to noise ratio (SNR) and decision making. Thus alternative ways need to be researched in order to improve the SNR and compare their performance based on the current second derivative method. Threshold selection algorithms need to be investigated and their stability evaluated. The new solution must allow for the use of the same pre and post processing in order to allow integration into the MKID instrument. This is defined in the following research questions.

Main research question:

“How can the current method to characterize and optimize large arrays of frequency-division multiplexed low temperature detectors be improved and enable it to work autonomously?”

Secondary research questions:

“How does the current second derivative method compare to another algorithm in terms of increasing the signal to noise ratio?”

“What algorithm would be able to provide the highest resolution in order to differentiate between closely overlapping resonances?”

“What decision making principle would be able to produce predictable and stable results in order to provide autonomy to the general method?”
2 SITUATIONAL & THEORETICAL ANALYSIS

The goal of this chapter is to describe the currently used Second Derivative method for resonance finding. This is performed with respect to the theoretical model, using its MATLAB® [13] algorithm.

2.1 Analysis of the Second Derivative Resonance Finding

The following analysis was obtained by studying the MATLAB second derivative algorithm and converting into Python 2.7. The choice of programming environment is discussed in the Research Design.

2.1.1 Pre Processing

During the pre-processing of the Second Derivative method, the spectral information is extracted from an input file. The information is divided into three columns representing: Frequency, Real Component and Imaginary Component. Using the real and imaginary components, the data is converted into one complex number, allowing for easier data manipulation by means of mathematical functions.

The amplitude is divided by its maximum in order to scale the data. The data is now represented by a maximum of 1 and a non-zero minimum. This is mainly used to obtain more stable areas of interest later on using a threshold.

The phase is unwrapped and smoothed using a moving average with a large window. The window is chosen to be large enough to smooth out all the resonances. This leaves only low frequency system drift and the phase delay, which is then subtracted from the original phase. The result leaves only the phase changes caused by the resonances. Figure 31 in Appendix A on page 53 is a flow chart of this step.

2.1.2 Signal to Noise Ratio Improvement

Although the phase delay has been removed, standing waves give a ripple in the amplitude. This step in the process is intended to increase the difference between the system ripple and resonances. Taking the derivative has the effect of dampening the low frequencies ripple and amplifying higher frequencies. The amplification is on both the resonances and the high frequency noise.

In order to dampen the latter, a low-pass top hat filter (moving average filter) is applied to the amplitude. The size of this filter is a preselected value, relative to the frequency step. Smaller frequency steps result in more high frequency noise. The result is an improved signal to noise ratio.

A preselected threshold is applied to the smoothed second derivative obtained from the previous step. The areas above the threshold represent the bottoms of the resonances. Even with overlapping resonances, a change in the amplitude’s curvature can be observed, which drops the second derivative below the threshold.

The result is segments of data that are referred to as Areas of Interest. These areas are extracted from the main information using indexes and afterwards related to the actual frequency, amplitude and phase. Figure 33 in Appendix A on page 53 is a flow chart of this step.

2.1.3 Resonance Detection

The resonance detection with the Second Derivative is straightforward. Within each area of interest, the minimum of the amplitude is taken as the resonance frequency. This does assume that each area of
interest spans only one resonance and thus in general represents a very small portion of the resonance. Figure 35 in Appendix A on page 53 is a flow chart of this step.

2.1.4 System Ripple Removal
Once the resonant frequencies are known, a preset area around each one is defined. A linear interpolation is performed over these areas in the amplitude and phase in order to create a signal consisting of the unwanted noise. Afterwards the amplitude is divided by this signal, resulting in amplitude populated only by resonances, which are also scaled with respect to the system ripple at their frequency. The phase is once again extracted and unwrapped. Figure 37 in Appendix A on page 54 is a flow chart of this step.

2.1.5 Circle Fitting
The resulting data are now ready to be fitted. Each pure resonance in the complex plane is represented by a circle. In the perfect case the phase does half a rotation, reaches the resonance peak at that point and continues back to 0. The resonances in the data are offset from the center of the complex plane and in order to obtain a correct phase each one needs to be centered. The fitting itself is performed by a Least-Squares Circle Fitting method, which converts the circle into a polynomial and solves it. The result of these procedure is shown in Figure 8.

The issues with circle fitting first arise from the number of data points around the resonant frequency that are used for fitting. Data points around the resonance are more dispersed and form a better circle, while those closer to the 0 phase are compact and distorted. Furthermore, overlapping resonances will distort each other around the resonant frequencies as well as requiring a more careful data point selection. A missed resonance in an overlap would result in a wrong circle fit. Figure 39 in Appendix A on page 54 is a flow chart of this step.
2.1.6 Post Processing

During the post processing the phase change of each resonance is divided by the frequency change. The maximum in this derivative multiplied by a quarter of the resonant frequency is the resonance’s Q factor. The Q factor is a dimensionless parameter that is used to relate the bandwidth to the resonant frequency as shown in Equation 1. With this known and the dip depth ($S_{21}^{min}$) obtained from the amplitude, the coupling and indicative Q factors are calculated using Equation 2 and Equation 3. With this information a resonance is considered characterized.

\[ Q = f_0 / \Delta f \]

*Equation 1 Relation between Q factor, resonant frequency and bandwidth.*

\[ Q = \frac{Q_c Q_i}{Q_c + Q_i} \]

*Equation 2 Relation between the Q factor, the coupling Q factor and the inductive Q factor.*

\[ S_{21}^{min} = \frac{Q_c}{Q_c + Q_i} \]

*Equation 3 Relation between the resonance dip depth, coupling Q factor and the inductive Q factor.*

This is essential because it allows for filtering of detections based on physical parameters. Detectors within an array are designed with specific Q factors and thus Q factors outside a certain range are considered false positives, or false negatives that were badly fitted, as seen in Figure 9. The reason for this is that the Q factor is calculated using frequency, amplitude and phase information and errors are clearly visible. Figure 41 in Appendix A on page 54 is a flow chart of this step.

*Figure 9 Example of a double resonance fitted incorrectly as one, in the LT028 Al Foil -42dBm. The blue dots represent the resonance, with the red being the resonant frequency. The green circle is the calculated fit.*
2.2 Method Overview

The spectral information received as an input is a complete transmission described by a phase and amplitude. During the Second Derivative method only the amplitude is used for the detection of resonances. This is done because in the phase the resonances have a smaller footprint relative to the system noise. The phase is required only during post processing. Post processing involves the circle fitting of each resonance in the complex plane in order to better observe changes in inductance as phase shifts of a centered resonance. Figure 10 is a summary of the full characterization algorithm that is currently implemented in MATLAB.

Microwave Kinetic Inductance Detector
Characterization Algorithm

Figure 10 Flow charts describing the steps used for the characterization of a MKID array starting from the raw spectral data and ending with individual resonance parameters. The algorithm was obtained from Second Derivative method, by dividing it into functional blocks.

2.3 Hypothesis

The section defining the current solution is the Signal to Noise Ratio Improvement, hence the selected name. The derivative is a measure of the change in amplitude with respect to frequency. A resonance caused by a MKID would result in a large change in amplitude. The second derivative represents the curvature of the change in amplitude, which is most pronounced at the very bottom of the resonance. Afterwards the threshold defines the minimum magnitude of the curvature. The biggest drawback in this method is that it just uses two properties of the resonance to detect them: the change with respect to frequency and the magnitude of the resonance. High frequency noise (with respect to the spectrum) has a strong response to differentiation as well. The algorithm is countering that effect using low pass filters, which get rid of the noise, but that also remove signs of overlapping resonances. Furthermore, the threshold will disregard shallow resonances because of their magnitude, disregarding their obvious shape.

The hypothesis is that an improved algorithm should focus on improving the signal to noise ratio. The thresholding would depend on the specifics of the new method. However the threshold should be related to a characteristic of the input spectrum, allowing for autonomy combined with predictable results.
3 CONCEPTUAL MODEL

For the purposes of this report the work performed will be separated into three sections. The work was performed sequentially:

- Research into potential methods for detecting MKID resonances
- Writing and testing of the original Second Derivative algorithm and the Continuous Wavelet Transform in Python 2.7
- Comparison between the performance of the two Python 2.7 algorithms

3.1 Selecting an alternative to the Second Derivative Method
The first part of the research involves finding a theoretical alternative. The problem with respect to this section can be defined as a one-dimensional detection algorithm. Detection algorithms are methods for finding events in a set of data, on the basis of knowing the event. For example face detection algorithms are one specific type of algorithms that are performed on two dimensional data (images), in order to obtain the location of a known event (a face).

The goal is to find a method that uses one-dimensional information from the frequency spectrums amplitude, and returns multiple center resonance frequencies. Methods that return the best match can potentially be modified to be usable within the context, since best is just a threshold passable by a single event.

3.2 Method Implementation
The Situational & Theoretical analysis describes how the Second Derivative method was rewritten from MATLAB to Python 2.7. This was used to provide better understanding into the challenges faced by the method. This new Second Derivative algorithm needs to be tested with respect to its MATLAB implementation, using a training data set. The only allowed difference in performance would be with respect to the required time to perform each algorithm, as that is dependent on the programming environment.

The new Continuous Wavelet Transform (CWT) solution is refined at this stage from its version during the first research step. This upgrade includes optimizing the structure and performance of the algorithm, to a point at which it is capable of fully characterizing MKID resonances. The final step is to prove that the CWT algorithm is capable of correctly finding resonances, within the training data set used by the Second Derivative, and obtain similar characterization results.

3.3 Algorithm Comparison
Assessment should allow for an objective comparison between the Second Derivative Algorithm and the Continuous Wavelet Transform Algorithm. Both algorithms aim to identify resonant frequencies in a frequency spectrum produced by MKIDs. During this process additional information regarding the data set is produced, which is to be considered secondary products of the algorithms.

Both algorithms require configuration relative to the variation in the input data. The stability of their configuration needs to be tested by using realistically changing data sets obtained from the same MKID array. Furthermore, their capabilities of handling noise and their resolution need to be tested without the influence of the calibration.
4 Research Design

4.1 Research tools
The initial tests of all methods are performed on data obtained from LT028, an A-MKID low frequency (350 GHz) test array designed with 880 detectors. The data is using readout power of -42dBm and it has a temperature load of 50K. These conditions result in most well defined resonances under real conditions. If a method is observed to require too much time to process the full set, a slice of the data will be taken. The slice is selected to include a good variety of overlapping and individual resonances.

Furthermore, all research is performed in Python 2.7 using the Anaconda package. Python is a programming language that offers qualities like robustness, flexibility, simplicity, reduced time to market and it is free. On top of its huge standard libraries Python offers additional mathematical tools designed in a syntax comparable to MATLAB [14].

4.2 Potential Methods
When researching ways to improve the signal to noise ratio, the first and most obvious choice is the use of filters. Filtering to some extent is already used in the Second Derivative method for dampening the high frequency noise. The idea is to test combinations between high and pass filters, to allow for an easier thresholding.

Weather radar and some airplane radar use an echo-location principle as their theoretical base. A signal is sent out and the receiver looks for the reflected signal from different objects. The signal that is received can be covered in white noise, shifted in phase, changed in frequency, time delayed etc. However, radar systems are still capable of recognizing their original signal and the difference contains information about the environment. The advantage of the radars is that those systems choose what type of signal to transmit and a common practice is transmitting a series of 4 pulses in different frequencies. This is a complex event, which even after all the transformation the signal experiences is still recognizable [15]. The methods implemented in radars, like matched filtering, can be investigated for finding MKID resonances. A prerequisite of the method is that the signal shape is known, which is true for a resonance. The amplitude and the width may change, but a human can still recognize them, which means that theoretically a suitable algorithm could also detect them.

Knowing the specific event in a signal also allows the use of methods like Wavelet Analysis, which allows both spectral and temporal analysis of a signal. Wavelet analysis is a relatively new concept (about 30 years old), and was developed as an alternative to the short time Fourier Transform (STFT) in order to overcome some resolution related problems. Wavelet analysis is the base from which several methods for processing signals have been derived. Those would be: the Continuous Wavelet Transform (CWT), the Discrete Wavelet Transform (DWT) and Multi-Resolution Analysis (MRA) [16]. These types of transforms find use for example in analyzing seismic data [17] and electrocardiograms [18].
4.3 Second Derivative and Continuous Wavelet Transform Implementation

The new implementation of the Second Derivative in Python needs to be tested with its MATLAB version. The best solution is to compare the two algorithms' final output on the same data set. The final output of the algorithms is the full characterization of all resonances. Statistical information is already used to assess a specific performance of MATLAB algorithm, so extracting the same information is an objective method to compare them. Since all parts of the method are used, deviations in the new algorithm will be most visible here. The statistical information is the following:

- Number of Detects
- Mean S21
- Standard deviation S21

Q factor statistics are highly susceptible to outliers so they will be shown using graphs.

Implementation of the Continuous Wavelet Transform will be performed for all stages of the characterization. Since this includes the final stages of circle fitting, the algorithm will be able to provide the same statistical information. For this comparison the CWT algorithm is allowed up two 15% deviation. This is considered the first objective comparison between the algorithms, however additional testing will be performed under less favorable real and simulated conditions.
4.4 Algorithm Comparison
The first step is to consider the nature of the algorithms. Every algorithm requires a certain number of assumptions in order to operate, which is considered calibration. It is done by adjusting the algorithm to work with a specific training data set to the required performance. Afterwards the performance of the algorithm is degraded by the difference between the training data and the actual data. One of the main problems encountered in the second derivative algorithm is the threshold selection, which represents the algorithm’s learning. The CWT algorithm has thresholds within it as well, with the most influential being the scales at which the CWT is performed. The research design must take this into consideration, since the calibration could lead to subjective results and conclusions.

Additionally in order to work with the new readout electronics an additional pre-processing step needed to be implemented. For more detailed description refer to Appendix D on 64.

4.4.1 Real Data
Real data present a great variation in the input signal, with of possible unknown elements and correlations. MKID arrays are designed to have a certain number detectors; however it is possible that due to manufacturing errors some of these detectors may be broken, and others may not behave as expected. Overlapping resonances can result in situations, in which even an instrument scientist has problems distinguishing them. Data sets taken from the same MKID arrays under different physical conditions provide a method for testing the algorithms stability. However, testing with real data is a necessary part of the testing procedure, as there is no other way to know how the algorithms would perform as part of a final setup.

The CWT algorithm is developed using a Vector Network Analyser (VNA) scan of a MKID array with a power setting of -42dBm. The same data set is used for the development of the Python implementation of the Second Derivative Algorithm. Thus it is considered the training data set for both algorithms’ calibration, and it could be a good place to obtain results under best conditions. The VNA scan includes data from higher and lower power settings, which present a controlled variation in only one parameter. In order to obtain some quantitative results this set is to be counted manually for resonances, which is then considered the real count. This allows for conducting a precision and recall test with a considerable number of relevant elements.

This test compares the known relevant elements that are the resonant frequencies obtained by eye, with the selected elements that are the resonant frequencies produced by the algorithms. Precision represents how many of the detections made by an algorithm are correct, with the rest being false positives. Recall represents how many of the detections made by eye are in the result produced by an algorithm, with the rest being false negatives.

Other real data sets available allow to test the algorithms’ performance with respect to the variation in the following physical characteristics: temperature of the detector array, temperature load on the detector array, different readout hardware and different MKID design. Since manually going through a data set is a time consuming task, the precision and recall would be conducted only on the original data set. The other possible variations would be tested with more relative comparison.

In order to obtain a fair perspective on the algorithms’ performance, without the influence of the calibration, both algorithms are going to use the calibration from the original data set. This includes smoothing windows, thresholds and other possible parameters within the algorithms.
Afterwards a second precision and recall test will be performed on a data set different from the learning "LT028 Al Foil". This will present a new variation in the position and behaviour of individual resonances, which will then be assessed with respect to changes in condition to the full array.

4.4.2 Simulated Data

In order to obtain the most objective results simulated data is required. When creating simulations it is possible to control every aspect of the data and relate that with the output results. This allows for characterising the algorithms based on their resolution, noise resistance, resonance Q factor variation and their calibration.

Furthermore, it is possible to establish the relationship between the calibration variation and the input variation of each algorithm. The two algorithms have a different number of parameters in their calibration, most of which are natural numbers. Additionally the Second Derivative has a real number for a threshold, and the simulated data can have up to four different real numbers as input. This results in an extreme multidimensional potential sampling area.

This problem is approached using a Monte Carlo analysis. This analysis substitutes a range of possible variables with a probability distribution. It calculates the results over and over, each time using a different set of random values from the probability functions. In this research a normal distribution is used for all of the variables. Algorithms are mainly assessed with respect to their performance in handling high frequency noise and distance between resonances.

\[
S_{21} = \frac{S_{21}^{\text{min}} + iX}{1 + iX}
\]

*Equation 4* Kinetic Inductance Detector resonance.

\[
X = 2Q \frac{f - f_0}{f_0}
\]

*Equation 5* Kinetic Inductance Detector normalized frequency.

Generating a complex resonance with the shape of a KID is done using Equation 4, where X is related to the Q factor and frequency by Equation 5. All simulated data sets will consist of two resonances and a small number of points, compared to the real frequency sweeps. The input signal and the calibration of the algorithms will be varied at the same time.

4.4.3 Additional Goals

Although locating the resonant frequency is the main output of the algorithms, the reasoning behind it is important as well. When considering the steps in both algorithms, the areas of interest in the data set are extracted, which can then be used for other data processing procedures. Some false positives in an algorithm can be overlooked if they can be easily filtered out in a subsequent. All pros and cons established from the results of each algorithm must be discussed. Last but not least, the required time to perform each algorithm must also be compared.
5  RESEARCH RESULTS

5.1  Potential Methods
High pass filters can be used to remove the system ripple, however during testing they proved to
dampen or completely remove low Q resonances. Furthermore overlapping resonances were also
missed.

A resonance during good conditions will resemble a Gaussian or Lorentzian function, which combined
with the matched filter’s implementation results in a simple low pass filter.

On the topic of wavelet transforms the first one to be tested was the Discrete Wavelet Transform. It is a
more complex version of the Short Time Fourier Transform (STFT). The DWT takes into consideration the
fact that high frequencies have a better time resolution and low frequencies have a better frequency
resolution. The DWT takes the signal and applies step by step complimentary low pass and high pass
filters based on a Mother Wavelet. The high pass information represents a large range of frequencies
with a high time resolution. The low pass information is subsampled by two and the filtering is
performed again. This can be done until the low pass returns a single specific frequency, spanning the
entire time domain.

DWT has found uses in encoding of information, compression of files and image processing. Figure 11
shows the effects of a DWT up to level 6. A resonance can be composed of a multiple of frequencies that
are spread out across all levels.

![Figure 11: Deconstruction of “LT028 Al Foil” amplitude using the Discrete Wavelet Transform. The original signal is shown on the top. Left hand side represents the result of the low-pass filtering and the right hand side of the high pass filtering. The DWT was performed using a Haar wavelet.](image)
Testing of Continuous Wavelet Transform (CWT) proved to have the best visible improvement of the SNR. Unlike the DWT, the CWT retains all spatial information for each frequency. The CWT performs a convolution with the continuous counterpart of orthogonal wavelets in the DWT at different scales. The result can be described as a 2 dimensional representation of how well the wavelet resembles the data, at a specific time and scale.

Wavelets are mathematical functions that begin at zero and by their end return back to zero. They may vary in complexity ranging from the simple Mexican Hat to a complex Shannon wavelet. Since the research is focused on the amplitude of the data the Mexican Hat wavelet was selected for the solution since it has a strong inverse response to the resonances. Figure 12 shows the selected wavelet in three different scales.

The Continuous Wavelet Transform performs a convolution between the input data and the wavelet at a given scale. The additional dimension in the transform’s result is obtained by changing the wavelet scale. Equation 6 shows the mathematical definition of the Continuous Wavelet Transform, where the transformed signal is a function of the translation ($\tau$) and scale ($s$) parameters.

$$\text{CWT}_x^\psi (\tau, s) = \Psi_x^\psi (\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t - \tau}{s} \right) dt$$

Equation 6 Continuous Wavelet Transform

The translation parameter is related to the location of the wavelet shifted through the signal, so once the transform is performed the size of this parameter will match the size of the input. The scale parameter in the wavelet analysis is similar to the scale used in maps. High scales represent a non-detailed global view (of the signal), and low scales correspond to a detailed view. In terms of frequency, low frequencies are high scales and high frequencies are low scales [16].

In the Python implementation of the CWT, the number of points used to represent wavelets increases with scale. This means that at scale 1, the convolution is performed between the input data and a 10 point wavelet. At scale 50 the convolution is performed with a 500 point wavelet, which increases linearly the processing time for each convolution. The linear increase in the convolution results in an exponential increase in the time required to perform the entire transform. Small data sets allow the use of high scales.
Although the actual values of the two dimensional data relate to the amplitude of the resonances, their shapes are only influenced by the wavelet. In the wavelet transform resonances have a shape and size that is independent of their amplitudes. However, when resonances are close to each other the higher scale wavelet fits better between them rather than on them. Both of these effects are represented in Figure 13 and Figure 14, where a highly detailed CWT transform is performed up to scale 80.

On top of this the resonance shape in the CWT is retained even in the presence of high frequency distortions as show in Figure 15. The shape of the resonances becomes sharper in the lower scales, but retains its size in the rest of the CWT.
5.2 Algorithm Implementation

This section covers the following topics:

- Test results that were used to prove that the new Python 2.7 implementation of the Second Derivative Algorithm to its MATLAB counterpart.
- Detailed explanation of how the Continuous Wavelet Transform method was used to create a Python 2.7 algorithm.
- Proof that the CWT algorithm is comparable to the previous one.

5.2.1 Second Derivative Testing

Results of the testing between the full implementation of the MATLAB and Python 2.7 algorithms in Table 1, as well as, Figure 16 and Figure 17 respectively. The MATLAB calculations for the Q factors take into account cryostats settings thus the resonances Q factor dependence with respect to frequency can be observed more clearly.

<table>
<thead>
<tr>
<th></th>
<th>MATLAB</th>
<th>PYTHON 2.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER OF DETECTIONS</td>
<td>797</td>
<td>797</td>
</tr>
<tr>
<td>MEAN OF S21</td>
<td>-4.255 dB</td>
<td>-4.257 dB</td>
</tr>
<tr>
<td>STD OF S21</td>
<td>1.363 dB</td>
<td>1.368 dB</td>
</tr>
</tbody>
</table>

Table 1 Comparison between MATLAB and Python 2.7 algorithms using the Second Derivative method. Statistics show are not dependent on the circle fitting procedure.

Figure 16 Q factors obtained using MATLAB. The readout power stated and difference is because of cryostat calibration is used.

Figure 17 Q factors obtained using Python 2.7 Second Derivative algorithm.
5.2.2 Continuous Wavelet Implementation

Using an 80 scale CWT on the full set of data, requires performing that number of convolutions on a data set of 150,000, and results in excessive processing time. Using trial and error it was decided that a CWT up to level 15 would be sufficient for this application, related to the data used in development. This is with respect to its step size and Q factors of the resonances in it.

The conversion into areas of interest is used by converting the data to a binary format and thresholding. All areas in which the CWT is above the threshold in a preset number of scales are considered to potentially contain resonances. The threshold is five times the standard deviation of the lowest CWT level. Resonances have the smallest possible footprint on the lowest scale of the transform so it is primarily composed of high frequency noise. Anything above five standard deviations above the noise is statistically relevant and is considered a signal, so that is why it is used as a threshold.

Continuous areas above the threshold in the frequency space, are considered areas of interest. The main goal is to find the resonance frequencies, which are now assumed to be contained within these areas and need to be pinpointed. The areas can include more than a few closely packed resonances that need to be identified. Close resonances start to merge together with increase in the CWT scale, so they need to be identified using the information contained in the lowest scales. On the other hand as stated earlier, the lowest CWT scales contain the high frequency noise of the signal. So another algorithm is applied to extract the resonant frequencies from the areas of interest using all scales of the CWT.

The advantage now present from having the areas of interest is that everything outside of them is considered to be the noise level. Using that now known noise level the shape of the resonances is extracted. Starting from the lowest scale, each step between the individual scales must be higher than 5 times the noise. The first point reached is considered the boundary between the noise within the area of interest and the resonance, which outlines the resonances. With the resonance outline, the lowest points in the outline are considered the resonant frequencies and the highest are the optimal division points between them. The steps described can be seen in Figure 18.
Figure 18 Example operation of the CWT algorithm, between 7.656 GHz and 7.661 GHz of MKID array LT028 Al Foil at -42dBm readout power. Top: Amplitude of input frequency spectrum. Middle: Extracted CWT areas defining the resonances. Bottom: Outline of the resonances in the CWT above the 5 times standard deviation threshold (blue lines) and the minima (red dots).
In summary the CWT algorithm is compared here to the model first described in Figure 10.

1. Pre Processing follows the same step as the Second Derivative method, except for the phase slope removal. The phase is processed in parallel with the amplitude at the ripple removal step. Figure 32 in Appendix A on page 53 is a flow chart of this step.

2. Signal to Noise Ratio is improved by performing a CWT on the amplitude. The division into area of interest is performed based on the areas of the CWT that are above 5 times the standard deviation found on the lowest CWT scale. Since areas of interest are significantly larger compared to the original solution, the areas are split into individual resonances in the next step. Figure 34 in Appendix A on page 53 is a flow chart of this step.

3. The shape of the resonances in the CWT information is extracted and local extremes in it are identified. In case of a single resonance, only a single minimum would be found. If there are multiple resonances it is assumed that each local minimum is a resonance and the local maxima between them are the split points. Figure 36 in Appendix A on page 53 is a flow chart of this step.

4. The areas of interest can be up to 300 kHz wide in frequency, which represents only the centre section of the resonance. In order to obtain the full resonance, the average width of the found areas is added to both of their sides. The additional areas include the shallow parts of the resonance. With the full resonance known again the same interpolation and division method is implemented afterwards to remove system ripple and phase delay, as was in the Second Derivative method. Figure 38 in Appendix A on page 54 is a flow chart of this step. Results of the performance of this step can be seen in Appendix B.

5. The circle fitting itself uses the same least square fit as the Second Derivative method. The least square fit method is highly reliant on using valid points from the resonances’ complex plain. The difference in this step is that the points used for fitting are a fraction of the area of interest around the resonant frequency. This means that in the case of a double, fewer points will be used for fitting on the side where the other resonance is. This has shown a tendency to provide better circle fitting. Figure 40 in Appendix A on page 54 is a flow chart of this step.

6. Once a circle has been fitted to a resonance the analysis follows the same steps as the original solution. Figure 42 in Appendix A on page 54 is a flow chart of this step.
5.2.3 CWT Proof of concept

By implementing the CWT method from beginning till end, it was proven that the new algorithm is capable of performing the resonance detection. Figure 19 shows the result of the entire conceptual model performed by the Continuous Wavelet Transform Algorithm. Thresholding the Q factors is the most reliable method for filtering out false positives, since it can be related to the array design. The other three graphs represent the performance of the fitting. The statistical data comparison can be seen in Table 2.

![Characterization Q factors CWT](image)

Figure 19 Full characterization of the LT028 Al Foil -42dBm MKID array, using the CWT algorithm.

Having proven the functionality of the newly developed solution, it can be compared to the current one. Although both algorithms are capable of converting a raw data set into a list of resonant frequencies with Q factors, the scope of this project’s comparison is until the resonance detection. The steps afterwards are taken into consideration only as part of the discussion.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Second Derivative</th>
<th>CWT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of finds</td>
<td>797</td>
<td>831</td>
</tr>
<tr>
<td>Mean of S21 dB</td>
<td>-4.255</td>
<td>-4.168</td>
</tr>
<tr>
<td>STD of S21 dB</td>
<td>1.363</td>
<td>1.359</td>
</tr>
<tr>
<td>Mean of Bandwidth</td>
<td>309.4 kHz</td>
<td>289.2 kHz</td>
</tr>
<tr>
<td>Mean of Q</td>
<td>21,784.86</td>
<td>25,292.34</td>
</tr>
<tr>
<td>STD of Q</td>
<td>5,940.590</td>
<td>8,391.303</td>
</tr>
<tr>
<td>Mean of Qc</td>
<td>68,274.89</td>
<td>76,139.90</td>
</tr>
<tr>
<td>STD of Qc</td>
<td>163,051.8</td>
<td>51,565.14</td>
</tr>
<tr>
<td>Mean of Qi</td>
<td>35,385.61</td>
<td>40,697.44</td>
</tr>
<tr>
<td>STD of Qi</td>
<td>8,604.405</td>
<td>13,895.01</td>
</tr>
</tbody>
</table>

Table 2 Statistical comparison between the characterized detects of the Second Derivative and CWT algorithms.
5.3 Algorithm Comparison

5.3.1 Original Data
The original data set is a Vector Network Analyser (VNA) scan using a 20 KHz frequency step, ranging from 4300.00 to 8014.30 MHz, in six different power settings. The total number of KIDs manufactured in this array is 880.

The manual counting was done on the -42dBm power setting. It is considered the best working set, for the clear resonances and low noise. During the counting the resonance frequencies were recorded in order to avoid miscounting and allow for a better comparison with different power settings. The end result is 850 resonances recorded, with only 2 occurrences of uncertain detections.

<table>
<thead>
<tr>
<th>Power Level</th>
<th>Continuous Wavelet Transform</th>
<th>Second Derivative</th>
<th>Eye Counting</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50dBm</td>
<td>828</td>
<td>484</td>
<td></td>
</tr>
<tr>
<td>-46dBm</td>
<td>814</td>
<td>793</td>
<td></td>
</tr>
<tr>
<td>-42dBm</td>
<td>831</td>
<td>797</td>
<td>850</td>
</tr>
<tr>
<td>-38dBm</td>
<td>822</td>
<td>805</td>
<td></td>
</tr>
<tr>
<td>-34dBm</td>
<td>828</td>
<td>807</td>
<td></td>
</tr>
<tr>
<td>-30dBm</td>
<td>816</td>
<td>750</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Original data set algorithm performance

Overall both algorithms perform within expectations and detect over 85% of all resonances, except for the drop in the Second Derivative’s performance at the low power settings. When looking at the results it can be seen that in all cases the missing resonances in the second derivative are related to the frequency. With higher frequencies lower Q factors are observed, as seen in Figure 20.

Figure 20 Amplitude overview scan of LT028 Al Foil at -50dBm power level, interpolated using CWT.
Deep resonances are still being detected by the Second Derivative method; however overlaps and low Q factor ones are being missed, as seen in Figure 21.

![Figure 21 Comparison of KID finding algorithms on the amplitude scan of LT028 Al Foil data set at high frequencies, interpolated using CWT.](image1)

Apart from this all other errors in both methods are caused by two or more closely overlapping resonances, as shown in Figure 22.

![Figure 22 Comparison of KID finding algorithms on the amplitude scan of LT028 Al Foil data set comparing the two algorithms with the eye detection. Interpolation was done using CWT.](image2)
5.3.2 Precision and Recall

Finally the statistical method of precision and recall is used to evaluate the performance of the algorithms. The “relevant elements” are considered to be detections made by eye. Despite being subjective based on the person counting, for the sake of these tests it is considered that a human with over six months of experience with KID resonances can offer the best possible detection.

It must be noted that only the -42dBm data set will be 100% correct for the precision and recall, since the resonance counting was performed on it. It is observed that with changes in the readout power all resonances shift, which can cause errors in the test. Two close detections from the algorithms can correspond to two real detections at one power level, but the shift can make both detections count towards one real detection, resulting in a false positive and the other one being missed causing a false negative. These cases are going to be overlooked in exchange for more data sets, since a manual count of an array can take up to 4 hours. Below Figure 23 shows the results of the precision and recall.

![Figure 23 Precision and Recall results of the LT028 Al Foil VNA power sweep.](image)

The results show that both algorithms have 0.9 on the precision, which means a low false positive count. The false positives are comparable between both algorithms and lowest at -42dBm. The Continuous Wavelet Transform algorithm shows better recall at all power levels, with the most drastic difference at -50dBm.
5.3.3 Visual Inspection
In order to obtain a better idea of the algorithms performance, they were tested using a 7 other MKID arrays. Of these 4 (including the LT028) are VNA scans and 4 are done using an Analog to Digital Convert (ADC) operating on a different principle. Further variations in the data sets include temperature loading, temperature of the array and an interlay different MKID design. The performance of the algorithms was tested at low, mid and high frequencies respective to the data set. Furthermore areas of interest, most commonly closely overlapping resonances, were looked into in order to show good and bad detections.

In order to try and objectively evaluate the algorithms the configurations of both algorithms haven’t been changed from LT028 -42dBm, 20 KHz frequency step. Below will be presented only a small selection of the data results. For all results check Appendix C.

5.3.3.1 External Load LT012
This data set has a 10 KHz frequency step and results in a sampling frequency twice the learning set. Although the data set includes the same readout power settings only the -42dBm is taken. This is done in order to provide the best possible conditions with respect to power. Overall detection numbers are shown in Table 4.

<table>
<thead>
<tr>
<th>External Load Temperature</th>
<th>Continuous Wavelet Transform</th>
<th>Second Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>50K ( Mirror )</td>
<td>852</td>
<td>847</td>
</tr>
<tr>
<td>77K ( Liquid Nitrogen )</td>
<td>842</td>
<td>830</td>
</tr>
<tr>
<td>300K (Room)</td>
<td>828</td>
<td>610</td>
</tr>
</tbody>
</table>

Table 4 Load Temperature Variation (LT012) Detection count comparison.

5.3.3.2 Base Temperature K123
The data available from K123 are 5 sets of VNA scans taken using different base temperatures. Those are 220mK, 340mK, 360mK, 380mK and 400mK. Although a different physical parameter is being changed the effects observed are similar to the effects caused by high temperature loading or low readout power. Furthermore the data sets have a frequency step of 5 KHz, which is 4 times faster than what the algorithms are calibrated for. For this test the 220mK, 340mK and 400mK sets were compared using the -42dBm readout power setting. Table 5 shows the overall results of these test.

<table>
<thead>
<tr>
<th>Array Base Temperature</th>
<th>Detection Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>220mK</td>
<td>Continuous Wavelet Transform</td>
</tr>
<tr>
<td>220mK</td>
<td>Second Derivative</td>
</tr>
<tr>
<td>340mK</td>
<td>1033</td>
</tr>
<tr>
<td>340mK</td>
<td>1000</td>
</tr>
<tr>
<td>400mK</td>
<td>3660</td>
</tr>
<tr>
<td>400mK</td>
<td>44</td>
</tr>
</tbody>
</table>

Table 5 Base Temperature Variation (K123) Detection count comparison using calibration for LT028 Al Foil -42dBm.
5.3.4 Precision and Recall on K123

5.3.4.1 Manual Counting and Offset Issue
In order to obtain the precision and recall for the K123 array data sets, manual counting was performed at -42dBm and 340mK. The result was 904 resonances of which at least 10 were questionable. First of all, the resonances obtained manually were compared to different settings in order to observe the general drift. Figure 24 shows the concerns that were discussed earlier with using a single manual count for multiple data sets with different readout power or temperature and Table 6 shows the implemented offset in order to solve the problem.

![Figure 24 Example of shift between different temperature and readout power in the interpolated amplitude of K123.](image)

The eye detection in the first resonance does not shift as considerably as is the case to its neighbouring double. If the current detections are used the result in this case would be two true positives, one false negative and one false positive. In order to decrease the effects of shifts an offset map was constructed. The values in the map would be added to the resonant frequency detected by eye and then used with the previous methodology. Although there are definitely going to be errors, bad shifts in cases like the left resonance would not result in an error in the precision and recall.

<table>
<thead>
<tr>
<th>mK/dBm [MHz]</th>
<th>220</th>
<th>340</th>
<th>360</th>
<th>380</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>0.42</td>
<td>-0.3</td>
<td>-0.44</td>
<td>-0.97</td>
<td>-1.68</td>
</tr>
<tr>
<td>-34</td>
<td>0.56</td>
<td>-0.11</td>
<td>-0.25</td>
<td>-0.75</td>
<td>-1.54</td>
</tr>
<tr>
<td>-38</td>
<td>0.72</td>
<td>0.03</td>
<td>-0.16</td>
<td>-0.8</td>
<td>-1.65</td>
</tr>
<tr>
<td>-42</td>
<td>0.84</td>
<td>0</td>
<td>-0.22</td>
<td>-0.92</td>
<td>-1.84</td>
</tr>
<tr>
<td>-46</td>
<td>0.89</td>
<td>-0.08</td>
<td>-0.3</td>
<td>-1.03</td>
<td>-2</td>
</tr>
<tr>
<td>-50</td>
<td>0.91</td>
<td>-0.16</td>
<td>-0.36</td>
<td>-1.14</td>
<td>-2.11</td>
</tr>
</tbody>
</table>

*Table 6 Eye detections offset map, with respect to readout power and base temperature.*
5.3.4.2 Overview of performance

Table 7 shows the detections count of each algorithm with respect to the selected variables of the input data. Although, not highly informative by itself, used with the precision and recall results it can give an objective comparison of the two algorithms. At first glance, it seems like the CWT detects do not drop below 96% of the actual count, whereas the second derivative goes below the 20% threshold.

<table>
<thead>
<tr>
<th>Continuous Wavelet Transform</th>
<th>Second Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>mK/dBm</td>
<td>220</td>
</tr>
<tr>
<td>-30</td>
<td>893</td>
</tr>
<tr>
<td>-34</td>
<td>936</td>
</tr>
<tr>
<td>-38</td>
<td>948</td>
</tr>
<tr>
<td>-42</td>
<td>912</td>
</tr>
<tr>
<td>-46</td>
<td>874</td>
</tr>
<tr>
<td>-50</td>
<td>878</td>
</tr>
</tbody>
</table>

Table 7 Number of detections by the Second Derivative and CWT algorithms, with respect to the base temperature and readout power of array K123.
5.3.4.3 **Precision and Recall**

The relevant elements are detections obtained manually combined with the average drift between readout power and base temperature. The end result is shown in the four sub plots in Figure 25.

![Figure 25](image-url)

*Figure 25 Precision and Recall comparison between the Continuous Wavelet Transform and Second Derivative detection algorithms relative to the readout power and base temperature of a Microwave Kinetic Inductance Array K123.*

In the case of more favorable conditions, seen in the top-left part of all graphs, both algorithms perform similarly enough to be considered equal. However with increase in noise, the algorithms’ results start to significantly differ in both precision and recall. The Continuous Wavelet Transform precision drops significantly below 0.3, whereas the Second Derivative manages to maintain a better precision. This can be explained, using the previous results, with the 2nd Deriv. starting to miss lower Q resonances, but it picks up noise as detections in higher Q ones. The CWT is more prone to pick up noise regardless of amplitude. This is representative of both algorithms’ breaking down, caused by bad configuration.

On the other hand the CWT maintains a reasonably high level of recall regardless of conditions. In summary errors increase with noise levels for both algorithms, but the CWT produces more false positives and the 2nd Deriv. more false negatives.
5.3.5 Simulated Data

Input Variation of Monte Carlo Analysis performed on the Second Derivative and Continuous Wavelet algorithms. The simulation is performed on a frequency scale of 10 MHz with a frequency step of 20 kHz.

Table 8 shows the variation space used during the simulation and Figure 26 shows the amplitude of a generated example signal on which the analysis will be performed. Table 9 and Table 10 show the variation in calibration parameters used for each of the resonance detection algorithms. The ranges are using uniform distribution (UnD) and linear distribution (LD) generated using random seed of 42.

![Monte Carlo Simulated Signal](image)

**Figure 26** Simulated signal consisting of two resonances, used as input for the Monte Carlo simulation.

A result is considered successful only if the algorithm locates both resonances. The total number of successful results is the following:

The Second Derivative has 613 successes out of 100,000 trials or 0.613%.

The CWT has 3,650 out of 100,000 trails or 3.650%.

### Table 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance between Resonances</td>
<td>0.01 – 1 MHz</td>
</tr>
<tr>
<td>High Frequency Noise level</td>
<td>0.01 – 0.5</td>
</tr>
<tr>
<td>S21 of resonance 1</td>
<td>0.01 – 0.9</td>
</tr>
<tr>
<td>S21 of resonance 2</td>
<td>0.01 - 0.9</td>
</tr>
<tr>
<td>Total</td>
<td>39,208,950 Possibilities</td>
</tr>
</tbody>
</table>

*Table 8 Input signal parameters and range sizes used during the Monte Carlo Simulation. UnD = Uniform Distribution.*

### Table 9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Points to Fit</td>
<td>5 – 25</td>
</tr>
<tr>
<td>Smoothing Window</td>
<td>5 – 20</td>
</tr>
<tr>
<td>Threshold</td>
<td>0.05 – 0.95</td>
</tr>
<tr>
<td>Minimum Length</td>
<td>2 – 5</td>
</tr>
<tr>
<td>Sampling Range</td>
<td>10 – 30</td>
</tr>
<tr>
<td>Total</td>
<td>2,540,160 Poss.</td>
</tr>
</tbody>
</table>

*Table 9 Second Derivative Algorithm calibration variation during the Monte Carlo Simulation. UnD = Uniform Distribution.*

### Table 10

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>CWT Scales</td>
<td>5-25</td>
</tr>
<tr>
<td>CWT Smoothing Freq.</td>
<td>1-10</td>
</tr>
<tr>
<td>CWT Smoothing Scale</td>
<td>1-10</td>
</tr>
<tr>
<td>Conversion Levels</td>
<td>1-(Max CWT Scale-2)</td>
</tr>
<tr>
<td>Levels Threshold</td>
<td>1-(Max CWT Scale-3)</td>
</tr>
<tr>
<td>Minimum Area</td>
<td>3-8</td>
</tr>
<tr>
<td>Smooth Array Length</td>
<td>1-5</td>
</tr>
<tr>
<td>Max Decision Level</td>
<td>1-(Max CWT Scale-2)</td>
</tr>
<tr>
<td>Total</td>
<td>2,932,776,000 Poss.</td>
</tr>
</tbody>
</table>

*Table 10 CWT Algorithm calibration variation during the Monte Carlo Simulation. UnD = Uniform Distribution. LD = Linear Distribution.*
The Second Derivative and CWT algorithms’ capability of handling high frequency noise in the input is shown in Figure 27 and Figure 28 respectively. In the Second Derivative results there are no successful results past the 10% amplitude of high frequency noise with respect to the signal.

The Second Derivative and CWT algorithms’ capability of successfully detecting close resonances in the input is shown in Figure 29 and Figure 30 respectively.
5.3.6 Calculation Time

Considering the complexity difference between the two algorithms the required time in order to perform each method needs to be considered to a certain extent. In the case of the CWT the transform itself is the most time consuming process and the calibration parameters decides on how many scales the calculation is performed. The Second Derivative algorithm is much more simple and straightforward with respect to implementation and the required time would increase only linearly with the size of the input.

In order to obtain some rough estimate during the calculation for the Precision and Recall of K123, the time required to load the data, calculate the CWT and calculate the 2nd Deriv. was taken. Table 11 shows the result. The calculations are performed over a wide range of data in order to exclude the data variation. Overall, performing a 15 scale CWT takes more than 4 times as long as performing a Second Derivative. Furthermore, the detections count seems to be more relevant in the CWT than the 2nd Deriv. time. The final implementation of the algorithms is considered only for a single data set so the end result is 31.1 seconds for CWT and 5.7 seconds for the 2nd Deriv.

<table>
<thead>
<tr>
<th>Time in Seconds</th>
<th>Time Required to Load All Six Power Sweeps</th>
<th>Time Required to Calculate CWT</th>
<th>Time Required to Calculate 2nd Deriv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>220mK Set</td>
<td>258.01</td>
<td>186.32</td>
<td>36.35</td>
</tr>
<tr>
<td>340mK Set</td>
<td>222.87</td>
<td>165.99</td>
<td>36.17</td>
</tr>
<tr>
<td>360mK Set</td>
<td>213.07</td>
<td>162.85</td>
<td>36.18</td>
</tr>
<tr>
<td>380mK Set</td>
<td>265.78</td>
<td>215.99</td>
<td>36.23</td>
</tr>
<tr>
<td>400mK Set</td>
<td>216.28</td>
<td>202.51</td>
<td>28.52</td>
</tr>
<tr>
<td>Average</td>
<td>235.20</td>
<td>186.73</td>
<td>34.69</td>
</tr>
</tbody>
</table>

*Table 11 Time comparison between CWT and 2nd Deriv. over the K123 Precision and Recall Tests.*
6 DISCUSSION

6.1 Potential Methods
Resonances have a basic shape compared to those used in radars. Radars implement a complex signal composed of a known set of sinusoids at different frequencies. Although it can be buried in noise the matched filter is capable of extracting it. The fact that the MKID resonance in the perfect case has a Gaussian shape makes it too simple to be used with a matched filter.

After testing with a variety of different wavelets it was decided that the very nature of the DWT was not suitable for improving the SNR of a resonance. The useful information is dispersed across the DWT, which each next level being distorted by the process itself.

MKID resonances in the Continuous Wavelet Transform created a large footprint, which showed consistent property with respect to their size. The two dimensional data presents new challenges in its extraction, however this also allows for many extraction techniques. Viewing the resonances in both scale and frequency allows to the use of more information in the decision making section of the code.

6.2 Implementation of current and new solutions
The Python implementation is capable of perfectly recreating the results obtained from the MATLAB version. This is since the procedure used in MATLAB can be copied step by step using equivalent Python methods.

During the Continuous Wavelet Transform implementation, the hardest part was extracting the resonant frequency form within the area of interest. Overlapping resonances in the Second Derivative method will result in two smaller uninterrupted areas, where as in the CWT method the areas would merge. This required the use of an additional algorithm to find the gaps between two close resonances in their single CWT. The local extreme method is highly dependent on having a smooth outline, with real values. Regardless the areas of interest extracted through the banalization of the CWT prove to pick up low Q resonances without being affected by high frequency noise or the system ripple.

The main reason in the statistical difference between the two algorithms comes from the fact that the CWT algorithm includes those low Q resonances. This is most correctly seen in the higher mean of $S_{21}^{\text{min}}$, since the dip calculation does not involve the circle fitting. The CWT uses a dynamic circle fitting based on the size of the resonances in the transform combined with a non-smoothed Q factor extraction, resulting in higher Q factor mean. Although the full characterization is not the focus of this comparison, the CWT contribution to circle fitting is considered to be a considerable improvement by itself.

6.3 Comparison between the current and new solutions
In order to answer the research questions, the two algorithms need to be compared objectively. Since it is considered that the detection of the resonant frequencies is the bottle neck in performance it was the main focus of testing.

Using real data and optimal calibration parameters the CWT algorithm was proven slightly superior to the Second Derivative, primarily because of its capability to detect low Q resonances. When dealing with closely overlapping resonances both algorithms’ success is dependent on the smoothing filters applied. The CWT detected 97.7% of all resonances in the learning data set and the Second Derivative 93.7%, which is an improvement of 4% or 50% fewer missed MKID resonances.
When the conditions of the data deteriorated from the original set without adjusting the calibration parameters the difference between the two algorithms was most clearly seen. The Second Derivative method at one point stops detecting areas of interest. The resonance is considered the minimum in each area, thus if no area is detected no resonance is. In the real data more noisy data was related to lower Q factor resonances. Errors in the second derivative in this case are caused by the shallow MKIDs that do not pass the threshold. The lowest threshold is dependent on the high frequency noise levels.

The CWT algorithm on the other hand, depends on local extremes in the outline of the CWT. The first stage of errors, for example when there are 150% more detects than expected, is caused within the individual areas of interest. The algorithm correctly detects the individual areas. However once it checks for multiple resonances within a single area, the outline smoothing filter before looking for local extremes is not able to fully remove the high frequency noise coming from the lowest CWT scales. This results in a noise resonance being incorrectly divided into multiple.

These observations are confirmed by the precision and recall test on the K123 data. Considering the previous observation made on the real data, colder base temperatures and higher readout powers result in higher noise levels. Both algorithms are showing the best performance at -38dBm and 340mK. If the precession and recall are considered with equal value, then the only area that the Second Derivative is superior is with the combination of high readout power and high temperatures. In that case resonances are deep but asymmetric. The asymmetry does not affect the second derivative’s success rate. The noise causes false positives in the CWT algorithm result and hence the drop in precision. The decrease in recall of the Second Derivative is entirely cause by the fact that the resonances are incapable of passing the threshold, yet even with that threshold false positives are still present.

The simulated test results offered an objective comparison between the two algorithms, by excluding the calibration parameter dependence. To begin with the CWT is using 1000 times more variable space for its calibration parameters than the Second Derivative, and despite that it archives more than 5 times the successful results. When the Monte Carlo simulation results are filtered based on the amplitude of high frequency noise, the biggest advantage of the CWT algorithm is observed. The successful CWT results follow an inverse logarithmic function and there are combinations in the calibration parameters that produce a correct detection all the way to the end of the tested range. The noise in the rightmost side of the scale represents Gaussian white noise with 50% the amplitude of the deepest possible resonance. On the other hand in the Second Derivative algorithm there is a limit in which the low pass filters can no longer suppress the noise. That limit is at 10% high frequency noise levels.

The second way the simulated results were used was to obtain insight into the two algorithms’ capabilities with respect to their resolutions. Neither algorithm is capable of detecting two resonances if their gap is smaller than 60 kHz, which in this case is 3 points. Comparison between the algorithms for this characteristic is inconclusive. This is because of the highly dispersed success rate and similar shapes. With the information of the CWT algorithm the following assumption can be made. When the bandwidth of two resonances no longer overlaps, the success rate of the algorithm will have a normal distribution.

During run time of both algorithms the circle fitting operation was being performed. Both algorithms perform an equal amount of circle fitting at the 220mK set. In the 400mK set the CWT algorithms performs 3 times as many circle fits and the Second derivative performs 2 times less. Taking this factor out of time calculations the CWT algorithm requires 6 times longer to perform.
7 CONCLUSIONS

The Continuous Wavelet Transform algorithm shows improvements in the detection of low Q factor Microwave Kinetic Inductance Detector resonances in the frequency spectrum. While operating with a predefined configuration by an operator this is the most significant difference between the two algorithms, resulting in 4% more successful detections of resonances.

With increase in high frequency noise the CWT algorithm starts generating up to 3 times more false positive detections, without change its configuration. This is not a result of the properties of the Continuous Wavelet Transform, but is attributed to the underlying method of separating overlapping resonances.

On the other hand the Second Derivative algorithm exhibits false negative detections, with deterioration of conditions. The primary reason for this is that combination of the low pass filtering and a predefined threshold ignore shallow resonances. Lowering any of these variables results in false positives, since the method is indifferent to the low frequency components of the resonances.

Under simulated conditions using a Monte Carlo Analysis, the underlying limitation of the Second Derivative method was discovered. Using a stronger low pass filter destroys the resonances to a point that taking the second derivative of the spectrum no longer improves the signal to noise ratio. The simulations showed that this point is when the amplitude of the high frequency noise reaches 10% of the maximum signal amplitude. In comparison it was proven that the CWT algorithm can theoretically handle up to 50% high frequency noise.

Furthermore, during the simulation the CWT algorithm had 5 times more successful results while using 1000 times more configuration variance. This proves that the method is capable of producing more stable results, if the configuration in its implementation can be related to properties of the input data.

In conclusion the Continuous Wavelet Transform method was proven to successfully improve all aspects of the currently used Second Derivative method. Additionally the information provided by the continuous wavelet transform can be used to improve the characterization of resonances past the detection of the resonant frequencies of the Microwave Kinetic Inductance Detectors. The only tradeoff is increased processing time and complexity, which can be improved with optimization of the algorithm.
8 Recommendations

The first area of improvement is correlating the configuration of the Continuous Wavelet Transform with physical and statistical characteristics of the input data set. The implementation currently uses a significant number of calibration variables that could potentially be related to the design of the MKID array used and physical conditions under which it is being tested. This can be done by a deeper understanding of the mathematical theory behind the transform or performing additional analysis using simulated data.

The weakest link in the implementation of the Continuous Wavelet Transform is the extraction of information from its two dimensional results. Using an alternative method for obtaining the resonance frequencies can result in more stable results. Furthermore, correctly identifying and characterizing overlapping resonances can potentially result in methods for their use in the final instrument.

Finally during the research stages a case was discovered in which the continuous wavelet transform is used on complex signals. This can be used to include the phase information of the frequency spectrum, resulting in even better performance of the algorithm [19].
REFERENCES


Appendix A

ALGORITHM FLOW CHARTS
Figure 31 Flow chart of the pre-processing performed in the Second Derivative algorithm. The input complex information is scaled with respect to its maximum value as well as the delay in the phase is removed.

Figure 32 Flow chart of the pre-processing performed in the Continuous Wavelet Transform algorithm. The input complex information is scaled with respect to its maximum value.

Second Derivative Algorithm
Improving Signal To Noise Ratio

2*10 log10 of Amplitude

2*10 log10 of Amplitude

Differentiate

Low-Pass Filter

Output

Figure 33 Flow chart of the signal to noise ratio improvement performed in the Second Derivative algorithm. This is the defining step of the algorithm, responsible for differentiating the signal from the noise.

Continuous Wavelet Transform Algorithm
Signal To Noise Ratio Improvement

Scaled Amplitude

Perform Continuous Wavelet Transform

Low Pass Filter

Convert to Unipolar

Two Dimensional Binary Map

Figure 34 Flow chart of the signal to noise ratio improvement performed in the Continuous Wavelet Transform algorithm. Separating the signal and noise is performed here for CWT.

Second Derivative Algorithm
Resonance Detection

Second Derivative

Selected Threshold

Scaled Amplitude

EXTRACT Areas above Threshold

Find Discontinuities

Areas without Resonance

Find Minimum within Range

Resonant Frequencies

Figure 35 Flow chart of the resonance detection performed in the Second Derivative algorithm. In this step a threshold is applied to the second derivative. All sequential areas above a minimum size are considered areas of interest.

Continuous Wavelet Transform Algorithm
Resonance Detection

HID Data Used And Required

Two Dimensional Binary Map

Sum Scale Axis and Threshold

Areas of Interest Index List

Find Discontinuities

Figure 36 Flow chart of the resonance detection performed in the Continuous Wavelet Transform algorithm. In this step sequential areas above a minimum x and y size are considered as areas of interest & paired with previous data.
Figure 37 Flow chart of the system ripple removal performed in the Second Derivative algorithm. The areas below the second derivative are reduced on their edge by a set value. The resulting signal is smoothed and interpolated at the gaps, where the resonances were. Dividing by the original data results in a signal clean of system ripple.

Figure 38 Flow chart of the system ripple removal performed in the Continuous Wavelet Transform algorithm. The average size of the areas of interest found is added to each side of the areas. The interpolation is performed in the same way as the Second Derivative method, with the signal outside the areas.

Figure 39 Flow chart of the circle fitting performed in the Second Derivative algorithm. In this step a least square circle fit is performed in the complex plane, using a preset number of points around every resonant frequency found. From the result a calibration parameter is generated.

Figure 40 Flow chart of the circle fitting performed in the Continuous Wavelet Transform algorithm. Using the local minimums in the CWT outline, the algorithm looks for resonances. If more than one are found, the area of interest is split at the local maximum of the outline between them. Circle fitting is performed on a portion of every area of interest. The complex data is corrected on the spot.

Figure 41 Flow chart of the post processing performed in the Second Derivative algorithm. Using the corrected phase each MKID is Characterized producing the end results in green.

Figure 42 Flow chart of the post processing performed in the Continuous Wavelet Transform algorithm. Using the new phase each MKID is Characterized producing the end results in green.
Appendix B

INTERPOLATION RESULTS
Using the newer readout an additional set is also present, which proved a considerable challenge for both algorithms. Since the overall performance of both algorithms in this case is unsatisfactory. This data set was used to point out the performance of the data interpolation using the CWT.

Figure 43 shows the amplitude of the VNA scan before interpolation (LT028 -42dBm Al Foil). The only action performed on this data is scaling, which allows for static configurations to operate independently of the amplitude to some extent. Overall, considering the variety of data looked at so far it is considered an easy to work with set. On the other hand Figure 44 shows a hard data set.

The scaling performed during the preprocessing does not improve the data. On top of that the data set has a frequency step of 3KHz, thus the highest sampling rate of all sets used. This causes high frequency noise to be picked up during the interpolation, visible in Figure 45. Furthermore, these KIDs are of a different design than the original data resulting in highly asymmetrical resonances, Figure 46.

Despite these problems the CWT interpolation is still capable of performing an excellent job, with a configuration for less than a quarter of the sampling rate used here.
Appendix C

TEST RESULTS USING DIFFERENT MKID ARRAYS
C.1 External Load Temperature LT012

Both algorithms perform well in the cases of 1< K and 77K. This is because of the low variation in resonance Q factors and the low noise levels. The Second Derivative begins to give false negatives when loaded at 300K and as observed before at higher frequencies. This is shown in Figure 47 and Figure 48.

![Figure 47 Interpolated amplitude of LT012 with a 77 K load.](image1)

![Figure 48 Interpolated amplitude of LT012 with a 300 K load.](image2)

It can also be observed that the resonances loaded at 300K have a sharper drop in Q factors, with respect to frequency. The higher frequency is noted to cause occasionally false positives in the Continuous Wavelet Transform method as shown in Figure 49.

![Figure 49 Example of false positives in the CWT and false negatives in the Second Derivative. Array LT012 at -42dBm and load temperature of 300K.](image3)

The resonances within the data with heavy temperature loading are still recognizable by eye. However if the effect of the 300K load temperature is combined with low readout power, it becomes impossible even for a human operator to distinguish.
C.2 Detector Base Temperature K123

At 220mK the resonances have high Q factors, but are overdriven resulting in a bad shape. This cause of errors is represented in Figure 50.

All detections in this case made by the second derivative are correct, despite the resonance shape. The CWT on the other hand has possibly up to four false positives. The single rightmost detections if true would be of an extremely shallow resonance, slightly distorted in this case by the interpolation. As for the triple on the left side, there is certainly at least one resonance there missed by the second derivative method. This can be concluded since if it was a single resonance the slope would be steeper, as for the real number that is outside the current expectations. Overall both algorithms perform flawlessly with single detections.

With the increase of the base temperature, the noise level is increased. That combined with the higher sampling rate results in an increased number of false positives from the 340mK data set. Although in the instance shown in Figure 51 it seems that the second derivative is struggling more, as a whole it can be assumed that the algorithms are equally bad.
As usual the second derivative misses a lower Q resonance. Although the placement and amount of false positives vary between the two algorithms, the end result is that the old calibration is starting to show its flaws for both algorithms. Yet again the problems are predominant in the higher frequencies, compared to the lower ones as seen in Figure 52.
This effect is even more visible with the highest base temperature set of 400mK. In that case the CWT detects over 3000 resonances, which obviously indicates a massive number of false positives. On the other hand the second derivative exhibits the opposite effect and misses almost all detections; anything below 20% detection rate can be considered well outside the requirements.

Figure 53 shows the amplitude overview of the situation. The few detections made by the second derivative are visible in the very beginning, however in the last ¾ of the data there is nothing. The K123 array has two other temperature settings not taken into consideration at this point and there is also information for the 6 readout power settings from before. This makes it a good candidate for another precision and recall, which requires manual counting of detects.

Since the CWT interpolation is used a lot of noise is being included as areas of interest, however despite the false positive interpolation still all relevant data are being included unlike the second derivative. This can be observed in greater detail in Figure 54.
C.3 Variation in Readout K126

In order to further vary the input data, the algorithms are tested using the output of the newest readout setup used, with which come different KID designs as well. The difference between designs can be seen in their response to readout power, base temperature and Q factor. The different readout also presents some discontinuities, known as stitching artefacts. Figure 55 shows the overview of the amplitude of the frequency spectrum of Array K126.

![Amplitude overview of K126 interpolated using CWT.](image)

The total detections for this array are 1050 for the Continuous Wavelet Transform and 990 for the Second Derivative. This data set has a frequency step of 10 KHz and low noise levels. Errors in this set are caused by artifacts left over from the preprocessing, like the one shown in Figure 57. On top of that very low Q factor resonances are being missed by the Second Derivative, such as the one in Figure 56.

Despite those small errors the other visible problem is false positives of the CWT that seem to be confined to the lower frequencies. Example of these occurrences are visible in Figure 58. It is possible that there are resonances there with very low Q factors or just noise. Unfortunately, in order to know for sure which one is, a different readout power or base temperature is required.
Figure 56 Example of low Q factor resonance in the interpolated amplitude of K126.

Figure 57 Example of a stitching error in the interpolated amplitude of K126.

Figure 58 Examples of false positive CWT detections in the K126 interpolated amplitude.
Appendix D

STITCHING OF FREQUENCY SWEEPS
An alternative readout device was delivered in April at the SRON Groningen MKID research lab. Data sets of additional arrays were available, but they were taken using the same readout electronics. The difference between the two readout electronics is that the new one uses multiple readout frequencies at once. The VNA scans use only one frequency in a sweep, so the new readout is 400 times faster. An algorithm is currently used, which is not effective at converting the data into a suitable format for the detection algorithms.

The readout electronics produces a sweep file that follows the structure shown in Table 12.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<tbody>
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<td>3950.01</td>
<td>Bin 1638</td>
<td>Real</td>
<td>Imag.</td>
<td>Bin 1665</td>
<td>Real</td>
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<td>Bin N</td>
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<td>Imag.</td>
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<tr>
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<td>Bin 1638</td>
<td>Real</td>
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<td>Bin 1665</td>
<td>Real</td>
<td>Imag.</td>
<td>Bin N</td>
<td>Real</td>
<td>Imag.</td>
</tr>
<tr>
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<td>Bin 1638</td>
<td>Real</td>
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<td>Bin 1665</td>
<td>Real</td>
<td>Imag.</td>
<td>Bin N</td>
<td>Real</td>
<td>Imag.</td>
</tr>
</tbody>
</table>

Table 12 Structure of a MKID readout electronics' sweep file. The left most column is the LO frequency, which is used to relate the bins of the readout to a real frequency. The red rectangles represent a single point in the frequency spectrum. The first value is the bin number, second is the real S21 transmission and the third is the imaginary S21 transmission. Using the LO frequency, the bin number, and configuration stored in another file, the spectral information needs to be reconstructed.

The sweep file of array K126 for example has 515 LO frequencies and 1000 bins for each frequency. This information needs to be reconstructed using Equation 7.

\[ f = \left( \frac{(Bin \ Number - 0.5) \cdot Readout \ Bandwidth}{2 \ Bits \ Used} \right) + LO \ Frequency \]

*Equation 7 Using sweep file and info file information to convert the fields in the file to a real frequency.*

The pure amplitude and phase graphs of array K125 are shown in Figure 60. The 515 LO frequencies are divided in 5 groups with the following frequencies:

- 3950.01 MHz
- 4773.99 MHz
- 5597.96 MHz
- 6421.94 MHz
- 7245.91 MHz

Within each group the LO frequency step is 10 kHz. When following the real frequency the data go through a single bin for every LO frequency in a group. Afterwards the process continuous with the next bin. Each LO frequency is an individual sweep for the readout. The sweeps are performed in a way that a set amount of points will overlap between individual sweeps. The process of stitching is to convert these individual sweeps and LO frequency groups into one continuous data set. Figure 59 shows these gaps in the phase and amplitude, the removal of which is the focus of the required pre-processing method.
Figure 59 Example of overlapping points between individual bins and the LO frequency groups. In the amplitude overlapping the divergences between points of neighboring bins is small, however the gap between frequency groups is significant. In the phase both types of overlaps have a considerable difference between them as well as discontinuities occur during phase wrapping when it passes $\pi$.

Figure 60 Pure Amplitude and Phase plot overview of the frequency spectrum of array K125 obtained using the new readout electronics. The 5 obvious colored arrays are different LO frequency zones in the readout. The first starts at 3950.01 MHz and the second at 4773.99 MHz. The space in between is covered by a 103 LO frequencies with a step of 10 kHz. The process is repeated until the full spectrum is covered.
The stitching method that was implemented first processes the bin stitching within a LO frequency group and afterwards performs the procedure between them. A problem encountered during the implementation was that there is a mismatch between the frequencies of one bin and the next. This results in a non-uniform frequency step in the stitched data, however that is not an issue.

In the amplitude the overlapping points are used to perform a linear interpolation so that the values can be matched to the same frequencies. Afterwards the algorithm locates the smallest difference between the points and uses that as an offset. The data from the next bin are multiplied by the offset. In the phase the sample principle is applied; however in this case the phase offset is added to the information from the next bin, resulting in the continuous signal that can now be used by the characterization algorithms.

![Graphs of Amplitude, Phase, and Frequency Step](image)

*Figure 61 Final product of the stitching pre-processing. The smallest difference between the points of two overlapping bins is taken. The amplitude of the second bin is multiplied by the difference, and in phase the offset is added. The resulting signal is non-uniform with steps no bigger than the LO frequency step at the stitching points.*